4.6 Isosceles, Equilateral, and Right Triangles

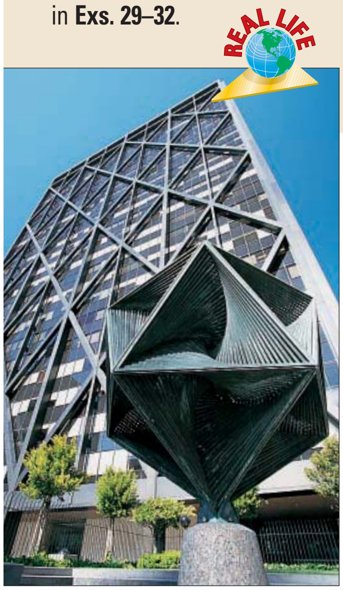
What you should learn

GOAL 1 Use properties of isosceles and equilateral triangles.

GOAL 2 Use properties of right triangles.

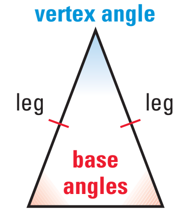
Why you should learn it

▼ Isosceles, equilateral, and right triangles are commonly used in the design of **real-life** objects, such as the exterior structure of the building in **Exs. 29–32.**



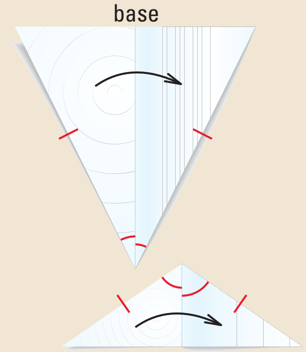
GOAL 1 USING PROPERTIES OF ISOSCELES TRIANGLES

In Lesson 4.1, you learned that a triangle is isosceles if it has at least two congruent sides. If it has exactly two congruent sides, then they are the legs of the triangle and the noncongruent side is the base. The two angles adjacent to the base are the **base angles**. The angle opposite the base is the **vertex angle**.



ACTIVITY Developing Concepts — Investigating Isosceles Triangles

1. Use a straightedge and a compass to construct an acute isosceles triangle. Then fold the triangle along a line that bisects the vertex angle, as shown.
2. Repeat the procedure for an obtuse isosceles triangle.
3. What observations can you make about the base angles of an isosceles triangle? Write your observations as a conjecture.



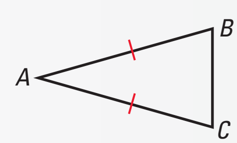
In the activity, you may have discovered the *Base Angles Theorem*, which is proved in Example 1. The converse of this theorem is also true. You are asked to prove the converse in Exercise 26.

THEOREMS

THEOREM 4.6 ***Base Angles Theorem***

If two sides of a triangle are congruent, then the angles opposite them are congruent.

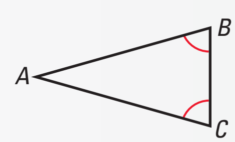
If $\overline{AB} \cong \overline{AC}$, then $\angle B \cong \angle C$.



THEOREM 4.7 ***Converse of the Base Angles Theorem***

If two angles of a triangle are congruent, then the sides opposite them are congruent.

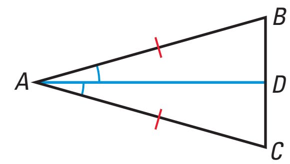
If $\angle B \cong \angle C$, then $\overline{AB} \cong \overline{AC}$.



EXAMPLE 1 *Proof of the Base Angles Theorem*

Proof

Use the diagram of $\triangle ABC$ to prove the Base Angles Theorem.



GIVEN ▶ $\triangle ABC$, $\overline{AB} \cong \overline{AC}$

PROVE ▶ $\angle B \cong \angle C$

Paragraph Proof Draw the bisector of $\angle CAB$. By construction, $\angle CAD \cong \angle BAD$. You are given that $\overline{AB} \cong \overline{AC}$. Also, $\overline{DA} \cong \overline{DA}$ by the Reflexive Property of Congruence. Use the SAS Congruence Postulate to conclude that $\triangle ADB \cong \triangle ADC$. Because corresponding parts of congruent triangles are congruent, it follows that $\angle B \cong \angle C$.

. . . .

Recall that an *equilateral* triangle is a special type of isosceles triangle. The corollaries below state that a triangle is equilateral if and only if it is equiangular.

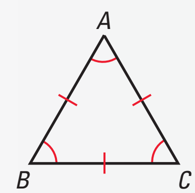
COROLLARIES

COROLLARY TO THEOREM 4.6

If a triangle is equilateral, then it is equiangular.

COROLLARY TO THEOREM 4.7

If a triangle is equiangular, then it is equilateral.

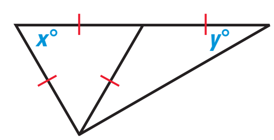


EXAMPLE 2 *Using Equilateral and Isosceles Triangles*

Using Algebra

a. Find the value of x.

b. Find the value of y.



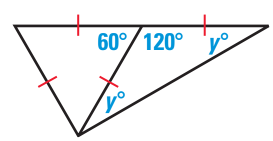
SOLUTION

a. Notice that x represents the measure of an angle of an equilateral triangle. From the corollary above, this triangle is also equiangular.

$3x^\circ = 180^\circ$ **Apply the Triangle Sum Theorem.**

$x = 60$ **Solve for *x*.**

b. Notice that y represents the measure of a base angle of an isosceles triangle. From the Base Angles Theorem, the other base angle has the same measure. The vertex angle forms a linear pair with a 60° angle, so its measure is 120°.



$120^\circ + 2y^\circ = 180^\circ$ **Apply the Triangle Sum Theorem.**

$y = 30$ **Solve for *y*.**

STUDENT HELP

INTERNET

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

GOAL 2 USING PROPERTIES OF RIGHT TRIANGLES

You have learned four ways to prove that triangles are congruent.

- Side-Side-Side (SSS) Congruence Postulate (p. 212)
- Side-Angle-Side (SAS) Congruence Postulate (p. 213)
- Angle-Side-Angle (ASA) Congruence Postulate (p. 220)
- Angle-Angle-Side (AAS) Congruence Theorem (p. 220)

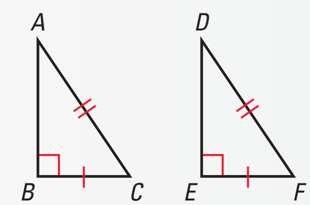
The Hypotenuse-Leg Congruence Theorem below can be used to prove that two *right* triangles are congruent. A proof of this theorem appears on page 837.

THEOREM

THEOREM 4.8 ***Hypotenuse-Leg (HL) Congruence Theorem***

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.

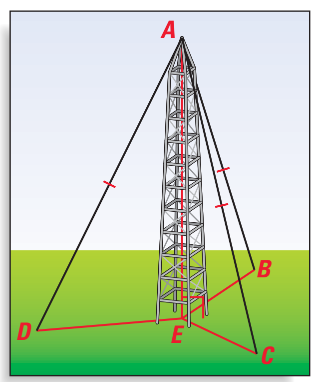
If $\overline{BC} \cong \overline{EF}$ and $\overline{AC} \cong \overline{DF}$, then $\triangle ABC \cong \triangle DEF$.



EXAMPLE 3 *Proving Right Triangles Congruent*

Proof

The television antenna is perpendicular to the plane containing the points B, C, D, and E. Each of the stays running from the top of the antenna to B, C, and D uses the same length of cable. Prove that $\triangle AEB$, $\triangle AEC$, and $\triangle AED$ are congruent.



GIVEN ▶ $\overline{AE} \perp \overline{EB}$, $\overline{AE} \perp \overline{EC}$, $\overline{AE} \perp \overline{ED}$, $\overline{AB} \cong \overline{AC} \cong \overline{AD}$

PROVE ▶ $\triangle AEB \cong \triangle AEC \cong \triangle AED$

SOLUTION

Paragraph Proof You are given that $\overline{AE} \perp \overline{EB}$ and $\overline{AE} \perp \overline{EC}$, which implies that $\angle AEB$ and $\angle AEC$ are right angles. By definition, $\triangle AEB$ and $\triangle AEC$ are right triangles. You are given that the hypotenuses of these two triangles, $\overline{AB}$ and $\overline{AC}$, are congruent. Also, $\overline{AE}$ is a leg for both triangles, and $\overline{AE} \cong \overline{AE}$ by the Reflexive Property of Congruence. Thus, by the Hypotenuse-Leg Congruence Theorem, $\triangle AEB \cong \triangle AEC$.

▶ Similar reasoning can be used to prove that $\triangle AEC \cong \triangle AED$. So, by the Transitive Property of Congruent Triangles, $\triangle AEB \cong \triangle AEC \cong \triangle AED$.

STUDENT HELP

Study Tip
Before you use the HL Congruence Theorem in a proof, you need to prove that the triangles are right triangles.

Guided Practice

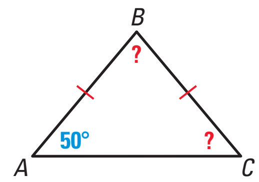
Vocabulary Check ✓

1. Describe the meaning of *equilateral* and *equiangular*.

Concept Check ✓

Find the unknown measure(s). Tell what theorems you used.

2.



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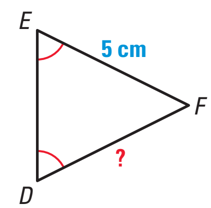

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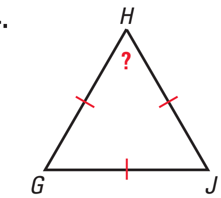
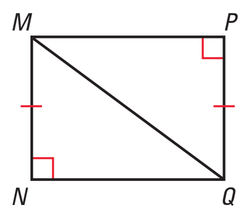


Skill Check ✓

Determine whether you are given enough information to prove that the triangles are congruent. Explain your answer.

5.



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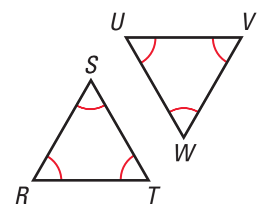

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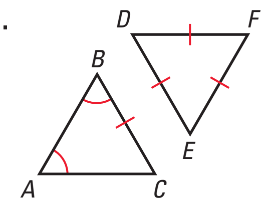


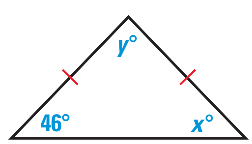
Practice and Applications

STUDENT HELP

Extra Practice to help you master skills is on p. 810.

USING ALGEBRA Solve for *x* and *y*.

8.



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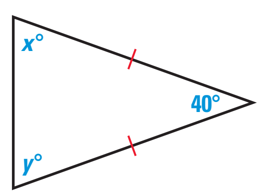

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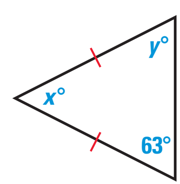
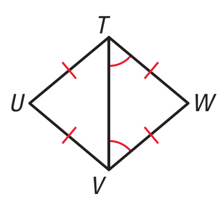
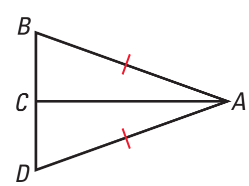


LOGICAL REASONING Decide whether enough information is given to prove that the triangles are congruent. Explain your answer.

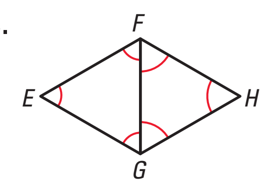
11.



12.



13.

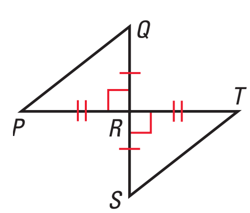


STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 26–28
Example 2: Exs. 8–10, 17–25
Example 3: Exs. 31, 33, 34, 39

14.



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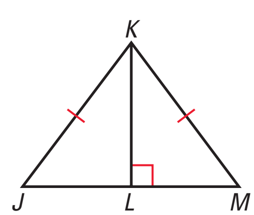

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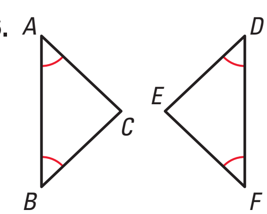
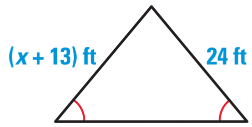
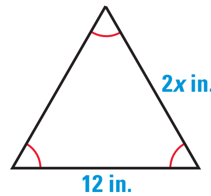


USING ALGEBRA **Find the value of x.**

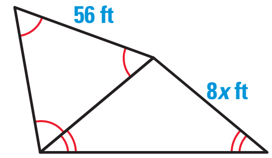
17.



18.

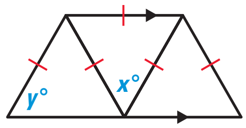


19.



USING ALGEBRA **Find the values of x and y.**

20.



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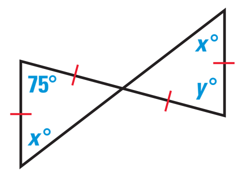

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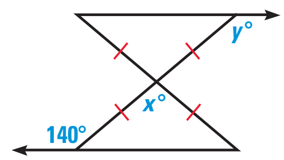

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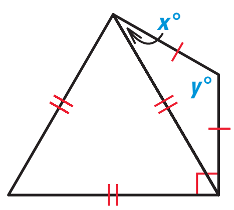

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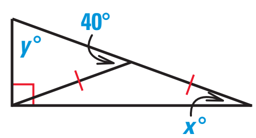

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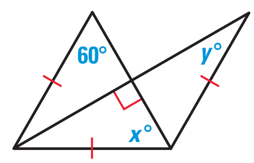


PROOF **In Exercises 26–28, use the diagrams that accompany the theorems on pages 236 and 237.**

26. The Converse of the Base Angles Theorem on page 236 states, "If two angles of a triangle are congruent, then the sides opposite them are congruent." Write a proof of this theorem.

27. The Corollary to Theorem 4.6 on page 237 states, "If a triangle is equilateral, then it is equiangular." Write a proof of this corollary.

28. The Corollary to Theorem 4.7 on page 237 states, "If a triangle is equiangular, then it is equilateral." Write a proof of this corollary.

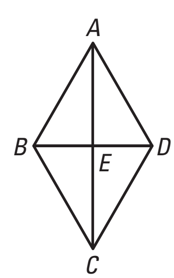
ARCHITECTURE **The diagram represents part of the exterior of the building in the photograph. In the diagram, $\triangle ABD$ and $\triangle CBD$ are congruent equilateral triangles.**

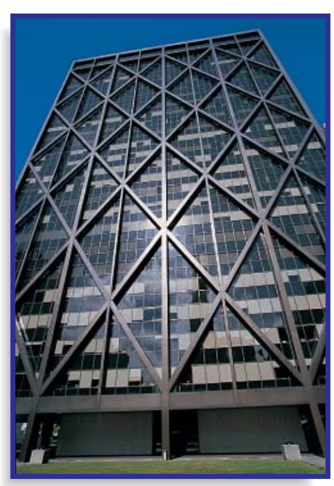
29. Explain why $\triangle ABC$ is isosceles.

30. Explain why $\angle BAE \cong \angle BCE$.

31. **PROOF** Prove that $\triangle ABE$ and $\triangle CBE$ are congruent right triangles.

32. Find the measure of $\angle BAE$.

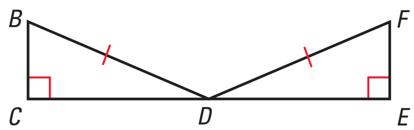




PROOF **Write a two-column proof or a paragraph proof.**

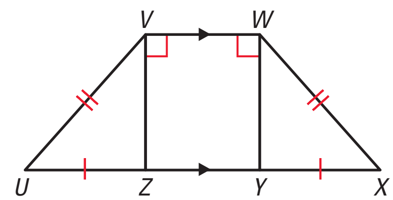
33. GIVEN ▶ D is the midpoint of $\overline{CE}$, $\angle BCD$ and $\angle FED$ are right angles, and $\overline{BD} \cong \overline{FD}$.

PROVE ▶ $\triangle BCD \cong \triangle FED$



34. GIVEN ▶ $\overline{VW} \parallel \overline{ZY}$, $\overline{UV} \cong \overline{XW}$, $\overline{UZ} \cong \overline{XY}$, $\overline{VW} \perp \overline{VZ}$, $\overline{VW} \perp \overline{WY}$

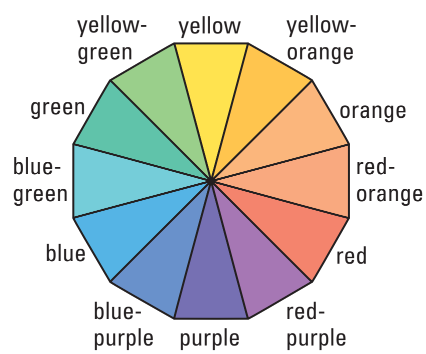
PROVE ▶ $\angle U \cong \angle X$



COLOR WHEEL **Artists use a color wheel to show relationships between colors. The 12 triangles in the diagram are isosceles triangles with congruent vertex angles.**

35. Complementary colors lie directly opposite each other on the color wheel. Explain how you know that the yellow triangle is congruent to the purple triangle.

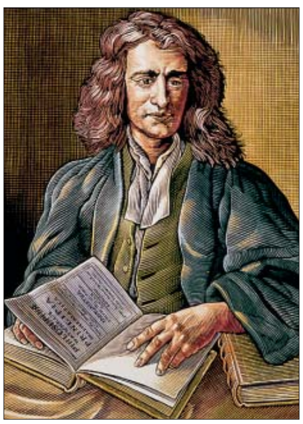
36. The measure of the vertex angle of the yellow triangle is 30°. Find the measures of the base angles.



37. Trace the color wheel. Then form a triangle whose vertices are the midpoints of the bases of the red, yellow, and blue triangles. (These colors are the *primary colors*.) What type of triangle is this?

38. Form other triangles that are congruent to the triangle in Exercise 37. The colors of the vertices are called *triads*. What are the possible triads?

FOCUS ON PEOPLE



REAL LIFE **ISAAC NEWTON** The English scientist Isaac Newton (1642–1727) observed that light is made up of a spectrum of colors. Newton was the first person to arrange the colors of the spectrum in a "color wheel."

INTERNET **APPLICATION LINK**
www.mcdougallittell.com

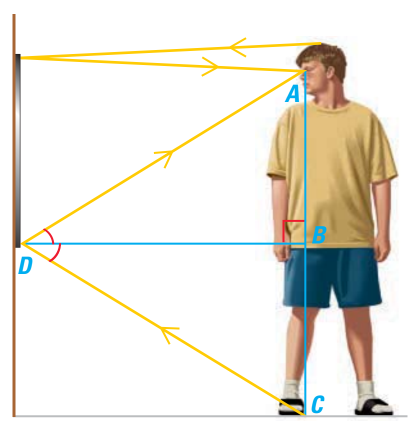
PHYSICS **Use the information below.**
When a light ray from an object meets a mirror, it is reflected back to your eye. For example, in the diagram, a light ray from point C is reflected at point D and travels back to point A. The *law of reflection* states that the angle of incidence $\angle CDB$ is equal to the angle of reflection $\angle ADB$.

39. GIVEN ▶ $\angle CDB \cong \angle ADB$
$\overline{DB} \perp \overline{AC}$

PROVE ▶ $\triangle ABD \cong \triangle CBD$

40. Verify that $\triangle ACD$ is isosceles.

41. Does moving away from the mirror have any effect on the amount of his or her reflection the person sees?



For a person to see his or her complete reflection, the mirror must be at least one half the person's height.

Test Preparation

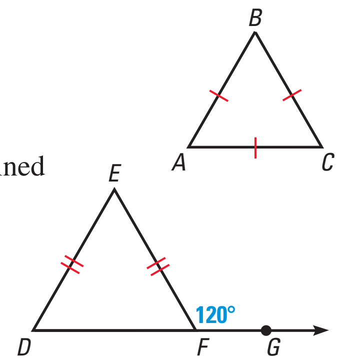
QUANTITATIVE COMPARISON In Exercises 42 and 43, refer to the figures below. Choose the statement that is true about the given values.

Ⓐ The value in column A is greater.

Ⓑ The value in column B is greater.

Ⓒ The two values are equal.

Ⓓ The relationship cannot be determined from the given information.

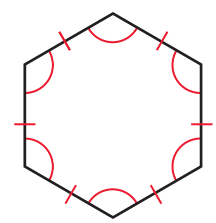


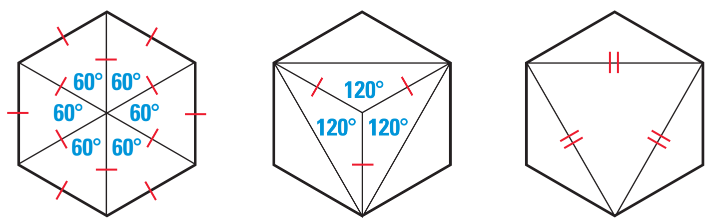
	Column A	Column B
42.	$\angle D$	$\angle EFD$
43.	$\angle B$	$\angle EFD$

★ Challenge

44. **LOGICAL REASONING** A *regular hexagon* has six congruent sides and six congruent interior angles. It can be divided into six equilateral triangles. Explain how the series of diagrams below suggests a proof that when a triangle is formed by connecting every other vertex of a regular hexagon, the result is an equilateral triangle.

Regular hexagon





EXTRA CHALLENGE
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MIXED REVIEW

CONGRUENCE Use the Distance Formula to decide whether $\overline{AB} \cong \overline{AC}$. (Review 1.3 for 4.7)

45. $A(0, -4)$
$B(5, 8)$
$C(-12, 1)$

46. $A(0, 0)$
$B(-6, -10)$
$C(6, 10)$

47. $A(1, -1)$
$B(-8, 7)$
$C(8, 7)$

FINDING THE MIDPOINT Find the coordinates of the midpoint of a segment with the given endpoints. (Review 1.5 for 4.7)

48. $C(4, 9), D(10, 7)$

49. $G(0, 11), H(8, -3)$

50. $L(1, 7), M(-5, -5)$

51. $C(-2, 3), D(5, 6)$

52. $G(0, -13), H(2, -1)$

53. $L(-3, -5), M(0, -20)$

WRITING EQUATIONS Line *j* is perpendicular to the line with the given equation and line *j* passes through point *P*. Write an equation of line *j*. (Review 3.7)

54. $y = -3x - 4; P(1, 1)$

55. $y = x - 7; P(0, 0)$

56. $y = -\frac{10}{9}x + 3; P(5, -12)$

57. $y = \frac{2}{3}x + 4; P(-3, 4)$